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POLARIZED PROTON-NEUTRON TOTAL CROSS SECTIONS FROM

PROTON-DEUTERON DATA

by

G. Alberi, M. Bleszynski, T. Jaroszewicz, and S. Santos

ABSTRACT

Simple expressions are derived for the polarized proton-deuteron total cross sections. Possibilities of extraction of the polarized proton-neutron cross sections from the proton-deuteron data are discussed.

Because recent measurements of the proton-proton (p-p) total cross section with polarized beam and target were successful,[1,2] and similar experiments are planned for the proton-deuteron case,[3] we are examining corrections to be applied to the raw deuteron data to obtain the polarized proton-neutron (p-n) total cross sections. These corrections were studied in detail for the unpolarized total cross sections[4,5] and were verified[6] using data obtained with neutron beams.

The general expression for the elastic-scattering amplitude of two particles of spin 1/2 and spin 1 can be obtained using parity and time reversal invariance.

$$\hat{F} = \hat{F}^{o} + \hat{\vec{F}}\cdot\vec{\sigma} \quad , \tag{1}$$

where

$$\begin{aligned} \hat{F}^{o} &= F^{o}_{o} + F_{y}\,\hat{J}^{o}_{y} + F^{o}_{xx}\,\hat{Q}_{xx} + F^{o}_{yy}\,\hat{Q}_{yy} \quad , \\ \hat{F}^{x} &= F^{x}_{x}\,\hat{J}_{x} + F^{x}_{xy}\,\hat{Q}_{xy} \quad , \\ \hat{F}^{y} &= F^{y}_{o} + F^{y}_{y}\,\hat{J}_{y} + F^{y}_{xx}\,\hat{Q}_{xx} + F^{y}_{yy}\,\hat{Q}_{yy} \quad , \text{ and} \end{aligned} \tag{2}$$

$$\hat{F}^x = F^x_x \hat{J}_x + F^x_{yx} \hat{Q}_{yx} \quad .$$

Here $\vec{\sigma}$ is the Pauli matrix of the external proton, $\hat{\vec{J}}$ is the deuteron spin, and $\hat{Q}_{ik} = 1/2 \left(\hat{J}_i \hat{J}_k + \hat{J}_k \hat{J}_i\right) - 2/3\, \delta_{ik}$.

As usual, the amplitude is defined in the deuteron brick-wall system[7] and z is along the average of the proton momentum, whereas y is orthogonal to the scattering plane. Actually, in the forward direction the brick-wall system coincides with the laboratory system and is along the incident-beam direction.

The missing terms change sign for either parity or time reversal transformation, where the x-, y-, and z-directions are defined in terms of the ingoing and outgoing momenta. Other terms are missing because they are not independent. When both beam and target are polarized along the i-th axis, the initial density matrix is

$$\hat{P} = \left(\frac{1}{2} - \frac{1}{2} P_i \hat{\sigma}_i\right) \left(\frac{1}{3} + \frac{1}{2} D_i \hat{J}_i + \frac{1}{2} A_i \hat{Q}_{ii}\right) , \qquad (3)$$

where P_i and D_i are the proton and deuteron polarizations along i, and A_i is the alignment of the deuteron defined[7] as

$$P_i = \mathrm{Tr} \left[\hat{\rho}\, \sigma_i\right],\ D_i = \mathrm{Tr} \left[\hat{\rho}\, \hat{J}_i\right],\ A_i = 3\ \mathrm{Tr} \left[\hat{Q}_{ii}\, \hat{\rho}\right] \quad .$$

The total cross section for the general density matrix, Eq. (3), becomes

$$\sigma_T = \frac{1}{\sqrt{\lambda(S, M^2, m^2)}}\ \mathrm{Im} \left(\mathrm{Tr} \left[\hat{F}\, \hat{\rho}\right]\right), \qquad (4)$$

where m and M are the nucleon and deuteron masses, s is the square of total c.m. energies of the system, and λ is a variable of relativistic kinematics.[8] From Ref. 6 for the transverse (y-direction) polarizations of the beam and the target,

$$\sigma_T = \frac{1}{\sqrt{\lambda(s, M^2, m^2)}}\ \mathrm{Im}\left\{\left(F^o_o + P_y D_y F^y_y + \frac{1}{6} A_y\, F^o_{xx} + \frac{1}{6} A_y P_y F^y_{yy}\right)\right\} , \qquad (5a)$$

and for the longitudinal (z-direction) polarizations,

$$\sigma_T = \frac{1}{\sqrt{\lambda(s,M^2,m^2)}} \; \mathrm{Im} \left\{ \left(F_o^o + P_z D_z F_z^z - \frac{1}{6} A_z \left(F_{xx}^o + F_{yy}^o \right) \right) \right\} \quad . \tag{5b}$$

For a purely vector-polarized deuteron target and a polarized beam, we can measure the $\Delta\sigma_L$ and $\Delta\sigma_T$ (Ref. 9) for proton-deuteron scattering, which are easily expressed as functions of the proton-deuteron spin amplitudes. These amplitudes are linear and bilinear expressions of the elementary c.m.* nucleon-nucleon amlitudes, in exactly the same way as the nonflip amplitude F_o^o (Ref. 4,5). Since the deuteron is larger in size than the nucleon, we can calculate the double-scattering integral[4,5] neglecting the t-dependence of the amplitudes. The result reads

$$\begin{aligned} \Delta\sigma_L = \sigma(\rightleftarrows) - \sigma(\rightleftarrows) &= \frac{4}{\sqrt{\lambda(S,M^2,m^2)}} \; \mathrm{Im}\left(F_z^z\right) \\ &= \frac{4}{\sqrt{\lambda(S,M^2,m^2)}} \left\{ \mathrm{Im} \left[\varepsilon_p(0) + \varepsilon_n(0) \right] \left(1 - \frac{3}{2} P_D \right) \right. \\ &+ \mathrm{Re} \left[2\,\varepsilon_p(0)\alpha_n(0) + 2\,\varepsilon_n(0)\,\alpha_p(0) - \alpha_p(0)\,\varepsilon_p(0) - \alpha_n(0)\varepsilon_n(0) \right] \\ &\left. \times R_L \frac{1}{4\pi\sqrt{\lambda(s,m^2,m^2)}} \right\} \quad , \end{aligned} \tag{6a}$$

and

$$\begin{aligned} \Delta\sigma_T = \sigma(\uparrow\uparrow) - \sigma(\downarrow\uparrow) &= \frac{4}{\sqrt{\lambda(s,M^2,m^2)}} \; \mathrm{Im}\left(F_y^y\right) \\ &= \frac{4}{\sqrt{\lambda(S,M^2,m^2)}} \left\{ \mathrm{Im} \left[\beta_p(0) + \beta_n(0) \right] \left(1 - \frac{3}{2} P_D \right) \right. \\ &+ \mathrm{Re} \left[2\beta_p(0)\,\alpha_n(0) + \beta_n(0)\,\alpha_p(0) - \alpha_p(0)\,\varepsilon_p(0) - \alpha_n(0)\,\alpha_n(0) \right] \\ &\left. \times R_T \frac{1}{4\pi\sqrt{\lambda(s,m^2,m^2)}} \right\} \quad , \end{aligned} \tag{6b}$$

*Actually the nucleon-nucleon spin amplitudes are calculated in the deuteron brick-wall frame, and they are connected to the c.m. amplitudes by a Wigner rotation.[10] However, in the forward direction the Wigner angle is zero.

where α, ε, and β are nucleon-nucleon c.m. amplitudes in the notation of Goldberger and Watson.[11] They are normalized as $\mathrm{Im}\,\alpha(0) = \sqrt{\lambda(s,m^2,m^2)}\,\sigma_T$; s is the square c.m. energy of the nucleon-nucleon system and p and n refer to proton-neutron scattering. The quantities R_L, R_T, and P_D can be expressed through the radial wave functions of the deuteron.[12,13]

$$R_L = \int_{\infty}^{\infty} dr\, r^{-2} \left[u(r) + \frac{1}{\sqrt{2}} w(r)\right]^2 ,$$
$$R_T = \int_{\infty}^{\infty} dr\, r^{-2} \left[u(r) + \frac{1}{\sqrt{2}} w(r)\right] \left[u(r) - \sqrt{2}\, w(r)\right] , \qquad (7)$$
$$P_D = \int_{\infty}^{\infty} dr\, w^2(r) \qquad \text{(D-wave percentage)} .$$

The values of R_L and R_T are given in Table I for three wave functions of the deuteron.[12,13] Also given are the D-wave percentages and the corresponding values for $\langle 1/r^2 \rangle$ for the S-wave renormalized to 1.

To take into account the t-dependence of the amplitudes, R_L and R_T must be expressed as q^2 integrals of the deuteron form factors.

$$R_L = \frac{1}{2\pi} \int d^2q \;\; \varepsilon(q^2)\alpha^{-1}(0)\alpha(q^2)\alpha^{-1}(0)$$
$$\times \int_0^{\infty} dr \left[u^2(r) - \frac{w^2(r)}{2}\right] j_0(qr) + \frac{1}{\sqrt{2}} \left[u(r)w(r) + \frac{w^2(r)}{2}\right] j_2\,(qr) ,$$

TABLE I

VALUES OF R_L, R_T, AND $\langle \frac{1}{r^2} \rangle$ FOR THREE DEUTERON WAVE FUNCTIONS

	R_L (fm^{-2})	R_T (fm^{-2})	P_D[a]	$\langle \frac{1}{r^2} \rangle$[b] ($fm^{-2}$)
McGee[13]	0.449	0.164	0.069	0.300
Reid S.C.[12]	0.382	0.157	0.064	0.265
Reid H.C.[12]	0.374	0.148	0.065	0.256

[a] D-wave percentage.

[b] For the S-wave renormalized to 1.

and

$$R_T = \frac{1}{2\pi} \int d^2q \quad \beta(q^2)\beta^{-1}(0)\alpha(q^2)\alpha^{-1}(0)$$

$$\cdot \int_0^\infty dr \left[u^2(r) - \frac{w^2(r)}{2} \right] j_0(qr) - \frac{1}{2\sqrt{2}} \left[u(r)w(r) + \frac{w^2(r)}{2} \right] j_2 \ (qr) \quad .$$

If we neglect higher order terms in the spin correlation expression C_{NN} in proton-proton scattering and we assume that the α and β phases do not vary with q^2, then q^2 behavior of $\beta(q^2)$ is related directly to the q^2 behavior of C_{NN}. For example,

$$\beta(q^2)/\beta(0) = \alpha(q^2)/\alpha(0) \cdot C_{NN}(q^2)/C_{NN}(0) \quad .$$

Although rough, this approximation allows qualitative testing of the R_L and R_T sensitivities to the amplitude t-dependence.

The results for 2 GeV/c are given in Table II; the same dependence as for $\beta(q^2)$ is assumed for $\varepsilon(q^2)$.[14]

We can express Eqs. (6a) and (6b) in terms of the polarized cross section[3] for the elementary processes p-p and p-n through relations of the type $2 \ \mathrm{Im} \left[\varepsilon_p(0)\right] = \sqrt{\lambda(s,m^2,m^2)} \cdot \Delta\sigma_L(0)$. The result[15] looks very similar to the famous Glauber formula[4] for the total cross sections. To do this however, we must neglect the real parts of the double-spin-flip amplitudes that could be large compared with the imaginary parts. Actually, the real parts were calculated by Grein and Kroll[16] using dispersion relations and the ratio $\rho_\beta^p = \left\{\mathrm{Re}[\beta_p(0)]\right\} / \left\{\mathrm{Im}[\beta_p(0)]\right\} \sim 6$ around 5 GeV/c. For a ratio $\rho_\alpha^p = \left\{\mathrm{Re}[\alpha_p(0)]\right\} / \left\{\mathrm{Im}[\alpha_p(0)]\right\} \sim -0.3$, we find that the Galuber formula is multiplied by a factor of 2.8. The ρ_β^n value, not known, must be calculated from the $\Delta\sigma_T$ values for proton-neutron scattering, extracted from deuteron data with the assumption $\rho_\beta^n = \rho_\beta^p$.

TABLE II

R_L AND R_T VALUES CALCULATED WITH t-DEPENDENCE OF THE NUCLEON-NUCLEON AMPLITUDES

	R_L (fm^{-2})	R_T (fm^{-2})
McGee[13]	0.401	0.115
Reid S.C.[12]	0.307	0.111
Reid H.C.[12]	0.306	0.105

It also can be determined self-consistently, for instance with an iterative procedure.

Although our discussion has been on pure Glauber theory, there are non-eikonal[17] corrections at intermediate energies ($P_{Lab} \leq 2$ GeV/c). The effect of these corrections is to modify the expressions for R_L and R_T [Eq. (6)] as follows.[18]

$$R_L' = \eta_L R_L = 1 + \int_{\infty}^{\infty} dr\, r^{-2} \left[u(r) + \frac{w(r)}{\sqrt{2}}\right] e^{2ikr} \left\{\frac{w(r)}{\sqrt{2}} \left[2 - 3\, j_o(kr)\right] e^{-ikr} - u(r)\right\}$$

and

$$R_T' = \eta_T R_T = 1 + \int_{\infty}^{\infty} dr\, r^{-2} \left[u(r) + \frac{w(r)}{\sqrt{2}}\right] e^{2ikr} \left\{\frac{w(r)}{2\sqrt{2}} \left[1 + 3\, j_o(kr) \quad e^{-ikr}\right] - u(r)\right\} .$$

The η_L and η_T values, listed in Table III for the incident-proton laboratory momenta, were calculated with the Reid soft-core wave function.[12] The non-eikonal corrections change mainly the R_T and R_L phases (by ∿5% for P_{Lab} = 1.7 GeV/c).

It was suggested recently that around 1.3 GeV the intermediate production of Δ_{33} plays an important role in proton-nucleus elastic scattering, but for a π^- exchange model for Δ production, the spin structure of the vertices $N\pi N$ and $N\pi\Delta$ is such that there is no contribution to the polarized cross section. At higher energies intermediate diffractive production should become important, but too little is known about the spin structure of diffractive production to be conclusive.

TABLE III

VALUES OF η_L AND η_T FOR SEVERAL INCIDENT-PROTON MOMENTA

K_{LAB} (GeV)	Re (η_L)	Im (η_L)	Re (η_T)	Im (η_T)
0.5	1.01	-0.15	1.12	0.35
1.0	1.00	-0.07	0.98	0.11
1.5	1.00	-0.04	0.99	0.06
2.0	1.00	-0.03	1.00	0.04
2.5	1.00	-0.03	1.00	0.04
3.0	1.00	-0.02	1.00	0.03

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